Design of RCC flexural members subjected to predominantly bending and low value of compression or tension

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R.J-Gu

30th oct 2020.

1.0Design of RCC flexural members subjected to predominantly bending moment and low value of compression or tension

1.1 Introduction:

The floor beam of a framed structure is subjected to predominantly bending moment associated with compression or tension depending on certain load cases. For design of such members, many engineers are using SP-16 charts and providing equal reinforcements in compression and tension zone. Though this approach is safe , it is illogical and uneconomical to provide equal reinforcements in tension and compression zone. The charts of SP-16 are meant for columns which are predominantly subjected to compression associated with some bending moment. It is not appropriate to make use of these charts for floors beams which are predominantly flexural members. Some engineers design these flexural members, only for bending moment and ignoring the effect of axial force .This approach is also wrong. This topic is very basic and frequently encountered. Nevertheles ,it is not addressed in the text books to the best of my knowledge and hence writing this article.

1.2 Procedure for design of flexural members subjected to predominantly bending moment and compression:

1.2.(a) Having assumed the cross section of the beam before the analysis, the design job is reduced to calculation of Asc/Ast based on the values of Mu & Pu from the computer output. We can choose to design such that the section is a balanced one so that maximum depth of concrete is utilized. The stress in compression steel is based on the strain and same can be referred from Sp-16, page 6. Using the equation $\sum$ M =0, about the tension reinforcement, we can get the compression to be contributed by the reinforcement and there from the value of Asc can be obtained. Subsequently, using the equilibrium equation $\sum$ H=0, we can find out the tension to be contributed by the reinforcement and there from the Value of Ast. Having got the values of Asc/Ast for different cases we can finalize the reinforcement considering other factors like ductility requirement/curtailment/standardization for the sake of detailing/availability etc. It can be noted that we can have multiple solutions for Asc/Ast depending on how we choose the depth of neutral axis. One such solution with Asc=Ast(similar to SP-16) can be found out by assuming the depth of neutral axis with different trials ranging from 0.15d to 0.2d(approximate).By resorting to the solution with Asc=Ast, concrete cross section will be effective only to the extent of around 20% where as by adopting the above design procedure as a balanced section,concrete cross sectional area will be effective to the extent of 48%.. Reversal in loading due to wind/seismic should not be an excuse for choosing the solution with Asc= Ast. Designing the section as a balanced one and providing the reinforcement based on all the cases,will result in considerable economy.The sample calculations are given in annexure1.

1.2.(b) When the beams are oversized it is sufficient to provide the reinforcement only in tension zone and design it as a singly reinforced beam and same is explained in annexure2.In other words theoretically reinforcement need not be provided in compression zone , when the resultant moment (equal to the applied moment plus the moment due to compression) is less than the limiting moment of

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resistance of the section. Calculation of Ast for such a case is given in annexure2.However, we should provide some reinforcement in compression zone from ductility and detailing considerations as per respective clauses.

1.2.(c) Further, given the beam size,P u & Mu, if we are to analyse a section the position of neutral axis should be found out using the equation ∑H=0 and then the moment capacity (Mu1) along with the associated force Pu can be found out by using the equation ∑M=0 about the tension reinforcement. Same can be compared against Mu to know the safety of the section. The sample calculations are given in annexure3.

1.3. Procedure for design of flexural members subjected to predominantly bending moment and tension:

1.3.(a) Procedure for calculation of Asc/Ast is given in annexure 4 which is exactly similar to the working done in annexure1,excepting the consideration for change in sign due to tensile force.

If we use SP-16 for the above case, it would be uneconomical and irrational.SP-16 gives the charts for bending and tension by totally ignoring the concrete area and considering only the reinforcement area. This approach is appropriate for the members which are subjected to predominantly tension and low value of bending where the neutral axis will lie outside the section i.e away from the top face.It is inappropriate to use the charts of SP-16 for a case where the member is predominantly subjected to bending and low value of tension.

SP-16 does not distinguish between the members subjected to bending associated with tension of high value or low value. Even if we decide to ignore the concrete completely (as done in SP-16),there is no point in using the charts since the same results can be obtained by the simple calculation i.e

Ast=Asc= (M/0.87fyxd1)+(T/0.87fyx2) which can be literally done at the back of an envelope.Use of this formula will give exactly the same result as that of Sp-16.(irrespective of concrete grade because concrete is totally infeffective.)

1.3.(b) When the beams are oversized it is sufficient to provide the reinforcement only in tension zone and design it as a singly reinforced beam and same is explained in annexure5.In other words theoretically reinforcement need not be provided in compression zone , when the resultant moment (equal to the applied moment minus the moment due to tension) is less than the limiting moment of resistance of the section. Calculation of Ast for such a case is given in annexure5.However,we should provide some reinforcement in compression zone from ductility and detailing considerations as per respective clauses.

1.3.(c) Further, given the beam size,T u & Mu, if we are to analyse a section the position of neutral axis should be found out using the equation ∑H=0 and then the moment capacity (Mu1) along with the associated force Tu can be found out by using the equation ∑M=0 about the tension reinforcement. Same can be compared against Mu to know the safety of the section. The sample calculations are given in annexure6.

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1.4 Conclusions & suggestions

1.4(a) The charts given in SP-16(for bending and compression or bending and tension) should not be used for members such as floor beams which are predominantly subjected to bending and low value of compression or tension. For these cases Asc/Ast should be calculated from strain diagram and equilibrium equations. The popular claim that LSD is more economical than WSD will become meaningless if we keep Asc=Ast as per sp-16

1.4.(b) These problems should be addressed in IS 456 and Sp16.Text books also should include problems on these topics. Senior engineers/teachers should teach these problems instead of teaching sp 16 which is self explanatory.

1.4(c) Software programmes used for the floor beams are giving unequal reinforcements for Asc/Ast. However these results should be validated by manual calculations as explained here.

1.4(d) The rectangular stress block can be used for concrete having an uniform ordinate of 0.4 fck over the depth of Xu.Further the stress strain curve for reinforcement can be simplified similar to that of mild steel in which case calculation of stress in reinforcement is very simple .i.e stress in reinforcement = (strain in reinforcement x Es) or 0.87fy whichever is lower.These concepts are widely used in US/USSR.Calculations for analysis/design for various cases based on the above concepts can be referred in my blog https://rgnsite.wordpress.com

 # About the author : Obtained my B.E & M.E degrees from Annamalai University in the years 1977 & 1979 respectively. I have nearly 3 years of teaching experience in Annamalai University and 33 years of professional experience in the engineering department of Larsen & Toubro construction group,Chennai.My Professional experience includes design of power plants,cement plants,material handling projects,steel plants and construction methods.

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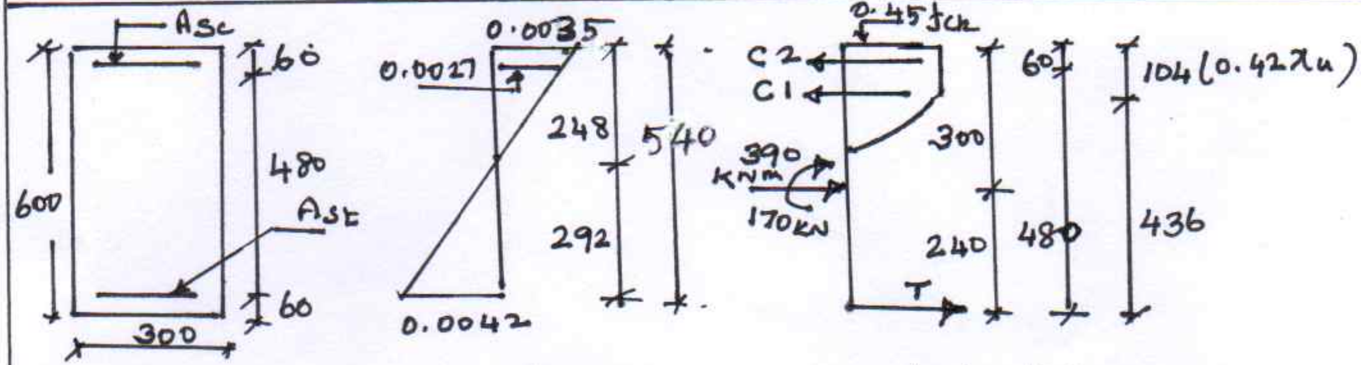
Annexure 1:

Floor beam of a framed structure is subjected to the following factored forces. $P_u = 170$ kN; $M_u = 390$ kNm; Beam size is 300 mm × 600 mm. Effective cover is 60 mm at top and bottom. Calculate Asc & Ast. Use M25 & Fy 500.



C.S. Strain diagram. Free body diagram.

Let us design this as a balanced section; $x_{ubal} = 0.46 \times 540 = 248$ mm;

$C1 = 0.36 \times 25 \times 300 \times \frac{248}{10^3} = 670$ kN; $\varepsilon_{sc} = \frac{0.0035}{248}(248-60) = 0.0027$;

$\therefore f_{sc} = 410$ N/mm². (Ref SP-16, P86, table A)

$f_{sc} - 0.45 f_{ck} = 410 - 0.45 \times 25 = 399$ N/mm²;

Use $\Sigma M = 0$, about tension reinforcement,

$390 + (170 \times 0.24) = (670 \times 0.436) + (C2 \times 0.48)$

$\therefore C2 = 289$ kN; $A_{sc} = \frac{289 \times 10^3}{399} = \underline{\underline{724 \text{ mm}^2}}$. (4-16 ф)

Use $\Sigma H = 0$; $170 + T = 289 + 670$; $\therefore T = 789$ kN;

$A_{st} = \frac{789 \times 10^3}{0.87 \times 500} = \underline{\underline{1814 \text{ mm}^2}}$; (4-25 ф)

Note: solution for Asc/Ast is not unique. We have multiple solutions for the same depending up on assumed value of 'x_u'. The solution with $A_{sc} = A_{st}$ given in SP-16 corresponds to the value of $x_u \leq 0.2d$.

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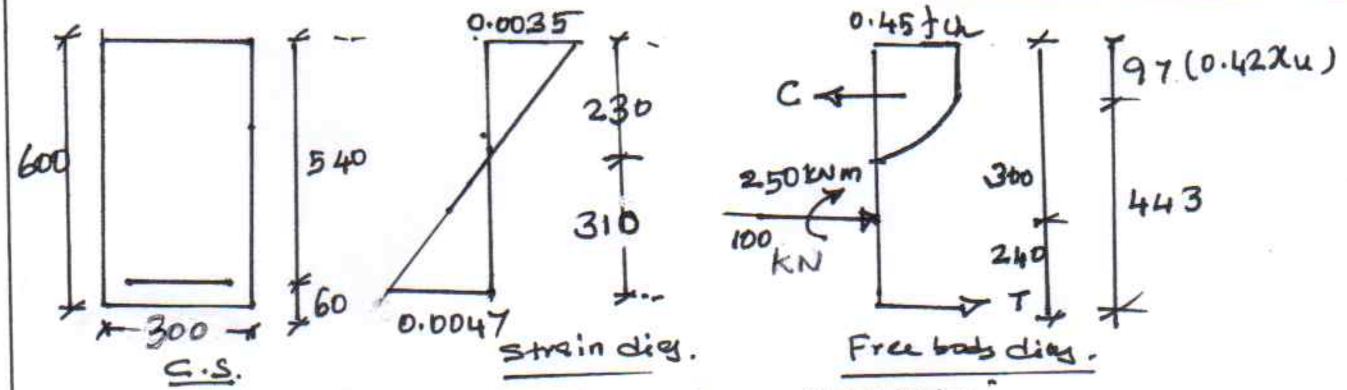
The beam cross section shown in the fig is subjected to factored forces $M_u = 250$ kNm & $P_u = 100$ kN. check whether it can be designed as singly reinforced and calculate the reinforcement. M25 & fy500 are used.



External moment = 250 + (100 × 0.24) = 274 kNm;

$M_{u\,limit} = 0.133 \times f_{ck} b d^2 = 0.133 \times 25 \times 300 \times \frac{540^2}{10^6} = 291$ kNm;

$M_{u\,limit}$ (291 kNm) > External moment (274)

It can be designed as a singly reinforced beam.

Based on trials, assume $x_u = 230$ mm;

$C = 0.36 \times 25 \times 300 \times \frac{230}{10^3} = 621$ kN;

Check for $\Sigma M = 0$ about tension reinforcement.

$\Sigma M = 274$ kNm; $\Sigma M = 621 \times 0.443 = 275$ kNm $\simeq$ 274 kNm

$\Sigma M = 0$ is fulfilled and assumed value of x_u is correct.

Use $\Sigma H = 0$; $100 + T = 621$; $\therefore T = 521$ kN;

$\varepsilon_{st} = 0.0047 > 0.0042$; $\therefore f_{st} = (0.87 \times 500)$ N/mm^2.

$A_{st} = \frac{521 \times 10^3}{(0.87 \times 500)} = \underline{\underline{1198 \text{ mm}^2}}$ (4-20 ϕ).

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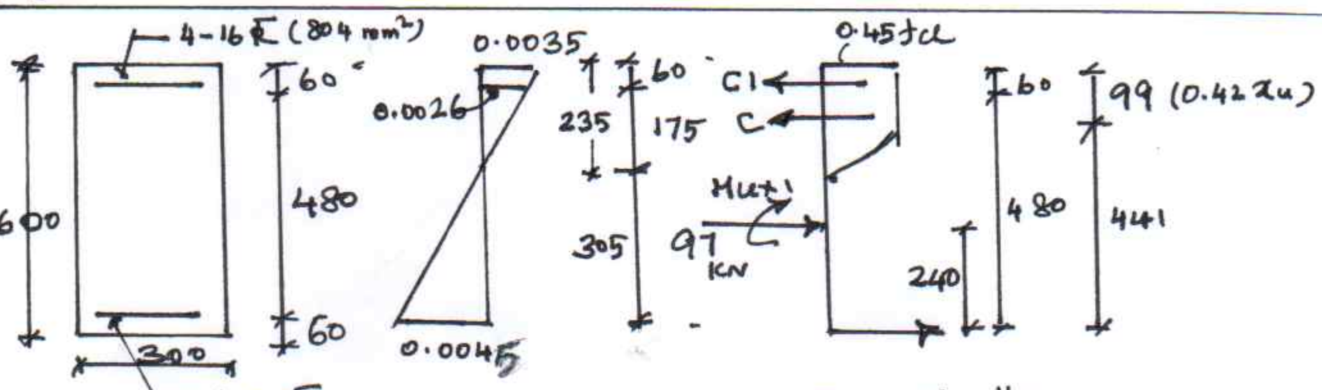
Annexure 3;

Given the factored forces $M_u = 350$ kNm & $P_u = 97$ kN, check safety of the section shown in the figure. M_{25} and f_y 500 are used.



C.S. Strain diag. Free body diag.

Let us calculate M_{ux1} (maximum moment capacity of the section along with the given axial force $P_u = 97$ kN).

Based on trials, assume $x_u = 235$ mm;

$\varepsilon_{sc} = \frac{0.0035}{235}(235-60) = 0.0026$; $\therefore f_{sc} = 404$ N/mm². (SP-16, table A)

$f_{sc} - 0.45 f_{ck} = 404 - 0.45 \times 25 = 393$ N/mm²;

C1 = Compression in reinforcement = $804 \times 393 \times \frac{1}{10^3} = 316$ kN;

C = compression in concrete = $0.36 \times 25 \times 300 \times \frac{235}{10^3} = 635$ kN;

$\varepsilon_{st} = \frac{0.0035}{235} \times 305 = 0.0045 > 0.0042$; $\therefore f_{st} = 0.87 \times 500 = 435$ N/mm²

T = Tension in reinforcement = $435 \times 1963 \times \frac{1}{10^3} = 854$ kN;

$\Sigma H_{\rightarrow} = 97 + 854 = 951$ kN; $\Sigma H_{\leftarrow} = 316 + 635 = 951$ kN; $\therefore \Sigma H = 0$;

The assumption made for neutral axis is correct.

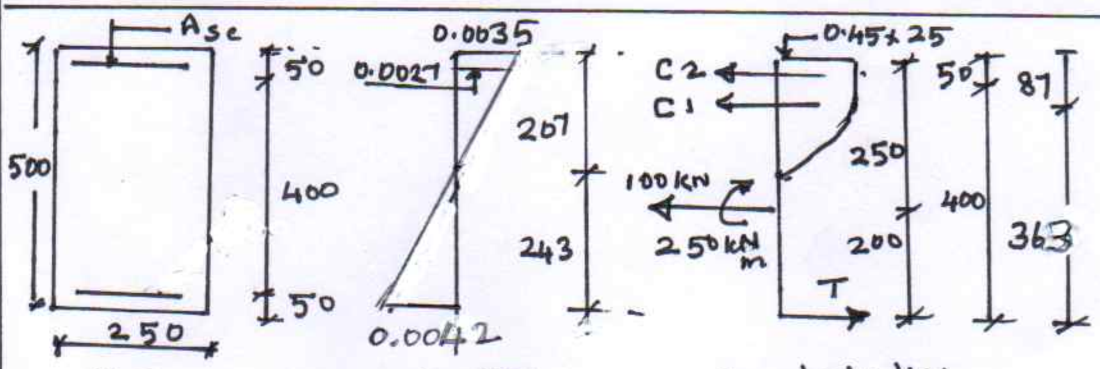
Use $\Sigma M = 0$, about tension reinforcement.

$M_{ux1} + (97 \times 0.24) = (316 \times 0.48) + (635 \times 0.441)$;

$\therefore M_{ux1} = 408$ kNm $> M_u (350$ kNm$)$

Hence the section is safe. R.J — [signature] 24th Aug 2020

Floor beam of a framed structure is subjected to the following factored forces $T_u = 100$ KN, $M_u = 250$ KNm; Beam size is 250 mm x 500 mm. Effective cover is 50 mm at top and bottom. Calculate A_{sc} & A_{st}. Use M_{25} & F_y 500;



C.S. Strain diag. Free body diag.

Let us design this as a balanced section; $x_{u\,bal} = 0.46 \times 450 = 207$ mm

$C1 = 0.36 \times 25 \times 250 \times \frac{207}{10^3} = 466$ kN; $\varepsilon_{sc} = \frac{0.0035\,(207-50)}{207} = 0.0027$;

$f_{sc} = 410$ N/mm²; (Ref SP-16, pg 6 table A) → based on ε_{sc}.

$f_{sc} - 0.45 \times 25 = 410 - 0.45 \times 25 = 399$ N/mm²;

Use $\Sigma M = 0$ about tension reinforcement.

$250 = (100 \times 0.2) + (466 \times 0.363) + (C2 \times 0.4)$

$\therefore C2 = 152$ KN; $A_{sc} = \frac{152 \times 10^3}{399} = \underline{\underline{381\ mm^2}}$; (4-12 ϕ)

Use $\Sigma H = 0$; $100 + 466 + 152 = T$; i.e $T = 718$ KN;

$A_{st} = \frac{718 \times 10^3}{0.87 \times 500} = \underline{\underline{1650\ mm^2}}$; (4-25 ϕ)

Solution for Asc/Ast given in SP-16 can be obtained by using the formula, $A_{st} = A_{sc} = \frac{M}{(0.87 f_y \times d')} + \frac{T}{(2 \times 0.87 f_y)}$;

(Concrete area is totally neglected.)

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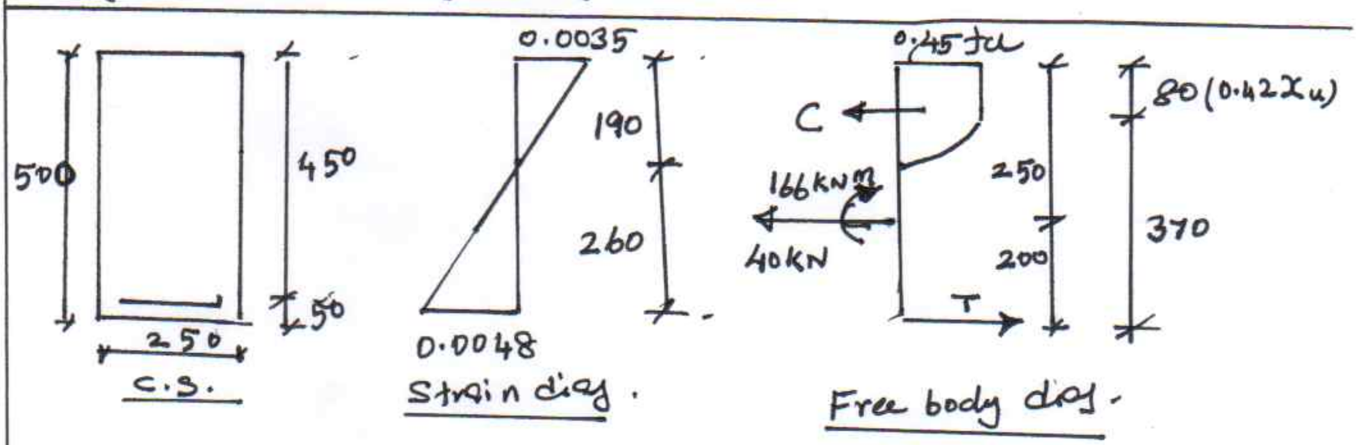
Annexure 5:

The beam cross section shown in the fig is subjected to factored forces $M_u = 166$ kNm & $T_u = 40$ kN. Check whether it can be designed as singly reinforced and calculate the reinforcement. M_{25} & $f_y 500$ are used.



External moment = $166 - (40 \times 0.2) = 158$ kNm

$M_{u\,limit} = 0.133 \times f_{ck}\, b d^2 = 0.133 \times 25 \times 250 \times \frac{450^2}{10^6} = 168$ kNm;

$M_{u\,limit}$ (168 kNm) > External moment (158 kNm)

It can be designed as a singly reinforced section.

Based on trials, assume $x_u = 190$ mm;

$C = 0.36 \times 25 \times 250 \times \frac{190}{10^3} = 428$ kN;

Check for $\Sigma M = 0$ about tension reinforcement.

$\circlearrowright \Sigma M = 158$ kNm; $\circlearrowleft \Sigma M = 428 \times 0.37 = 158$ kNm; $\therefore \Sigma M = 0$

The assumption made for neutral axis depth is correct.

Use $\Sigma H = 0$; $40 + (428) = T$; $\therefore T = 468$ kN;

$\varepsilon_{st} = 0.0048 > 0.0042 \;\therefore\; f_{st} = (0.87 \times 500)\ N/mm^2$.

$A_{st} = \frac{468 \times 10^3}{(0.87 \times 500)} = 1076\ mm^2$. (4-20$\phi$)

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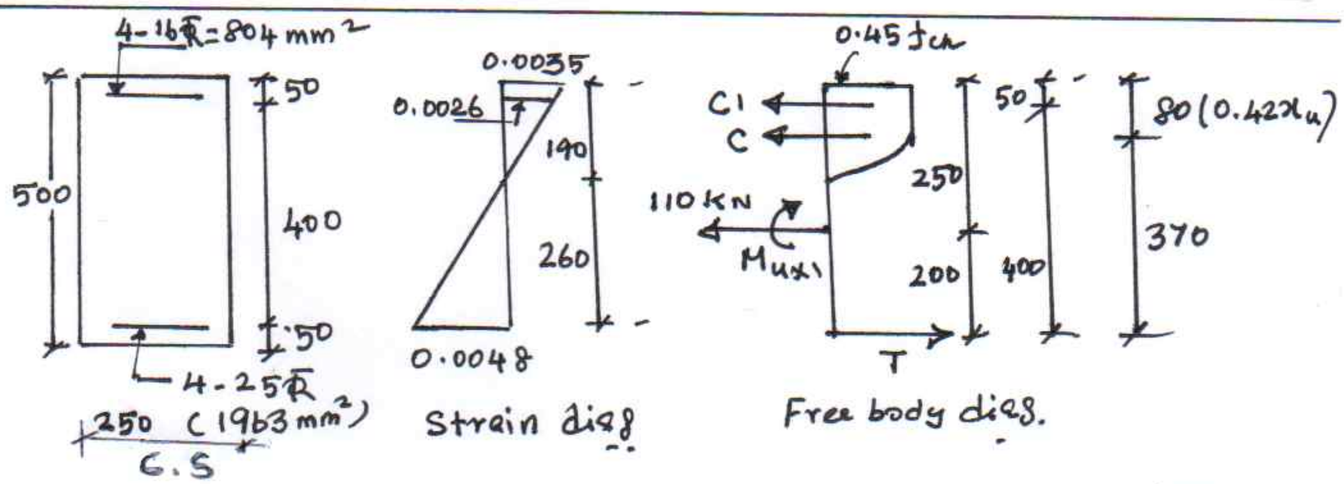
Given the factored forces $M_u = 280$ kNm & $T_u = 110$ kN

Check safety of the section shown in the fig. Use M_{25} & $f_y 500$;



Let us calculate M_{ux1} (maximum moment capacity of the section along with the given axial force $T_u = 110$ kN).

Based on trials, assume $x_u = 190$ mm;

$\varepsilon_{sc} = \frac{0.0035}{190}(190-50) = 0.0026$; ∴ $f_{sc} = 404$ N/mm². (SP-16, table A)

$f_{sc} - 0.45 f_{ck} = 404 - (0.45 \times 25) = 393$ N/mm²;

C1 = compression in reinforcement = $804 \times 393 \times \frac{1}{10^3} = 316$ kN;

C = compression in concrete = $0.36 \times 25 \times 250 \times \frac{190}{10^3} = 428$ kN;

$\varepsilon_{st} = \frac{0.0035}{190} \times 260 = 0.0048 > 0.0042$; ∴ $f_{st} = 0.87 \times 500 = 435$ N/mm²

T = Tension in the reinforcement = $435 \times 1963 \times \frac{1}{10^3} = 854$ kN;

$\sum H_{\leftarrow} = 428 + 316 + 110 = 854$ kN; $\sum H_{\rightarrow} = 854$ kN; ∴ $\sum H = 0$

The assumption made for neutral axis is correct.

Use $\sum M = 0$, about tension reinforcement.

$M_{ux1} = (110 \times 0.2) + (316 \times 0.37) + (428 \times 0.4) = 310$ kNm;

M_{ux1} (310 kNm) > M_u (280 kNm)

Hence the section is safe.

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A_{sc}/A_{st} for Beams subjected to M_u & P_u or T_u (of small value) i.e Bending is predominant

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M_u & P_u

$\left[M_u + P_u\left(d - \frac{D}{2}\right)\right] < M_{lim}$ ∴ Design as singly reinforced	$\left[M_u + P_u\left(d - \frac{D}{2}\right)\right] > M_{limit}$ ∴ Design as doubly reinforced
$A_{st} = \frac{(0.36 f_{ck} b x_u) - P_u}{(0.87 f_y)}$	$A_{sc} = \frac{\left[M_u + P_u\left(d - \frac{D}{2}\right)\right] - M_{lim}}{(0.8 f_y \times d_1)}$
A_{st} Calculated is higher than the theoritical requirement.	$A_{st} = \frac{\left[(0.36 f_{ck} b x_u) + (0.8 f_y A_{sc}) - P_u\right]}{(0.87 f_y)}$

M_u & T_u

$\left[M_u - T_u\left(d - \frac{D}{2}\right)\right] < M_{limit}$ ∴ Design as singly reinforced	$\left[M_u - T_u\left(d - \frac{D}{2}\right)\right] > M_{limit}$ ∴ Design as doubly reinforced
$A_{st} = \frac{(0.36 f_{ck} b x_u) + T_u}{(0.87 f_y)}$	$A_{sc} = \frac{\left[M_u - T_u\left(d - \frac{D}{2}\right)\right] - M_{lim}}{(0.8 f_y \times d_1)}$
A_{st} calculated is higher than the theoritical requirement.	$A_{st} = \frac{\left[(0.36 f_{ck} b x_u) + (0.8 f_y A_{sc}) + T_u\right]}{(0.87 f_y)}$

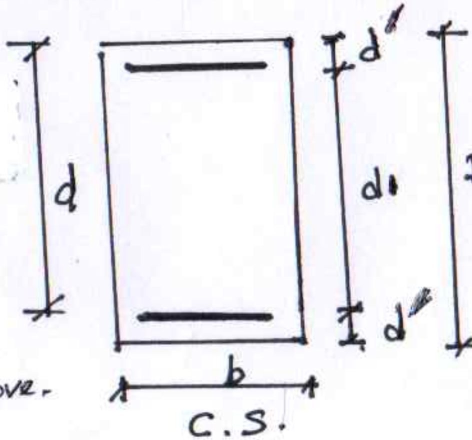
Assumptions:

Net stress in compression steel varies from $0.78 f_y$ to $0.82 f_y$ depending on factors like d', f_y, f_{ck}. Here it is assumed as $0.8 f_y$.

Section is designed as a balanced one. $x_u = 0.46d$, $f_y = 500 N/mm^2$, $M_{lim} = 0.133 f_{ck} b d^2$;

A_{st} as a singly reinforced section is calculated for a higher moment along with the axial force.

Never omit the axial force. Nor provide $A_{sc} = A_{st}$; Don't use SP-16 for above.



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